

1 [数学 I p12 練習9]

- (1) $3x^2(3x^2 - 5x + 2)$
 $= 3x^2 \cdot 3x^2 + 3x^2 \cdot (-5x) + 3x^2 \cdot 2$
 $= 9x^4 - 15x^3 + 6x^2$
- (2) $(x^2 - 2xy - 3y^2)(-xy^2)$
 $= x^2(-xy^2) - 2xy(-xy^2) - 3y^2(-xy^2)$
 $= -x^3y^2 + 2x^2y^3 + 3xy^4$
- (3) $(x^3 + 3x^2 - 4)(x - 2)$
 $= (x^3 + 3x^2 - 4)x + (x^3 + 3x^2 - 4) \cdot (-2)$
 $= x^4 + 3x^3 - 4x - 2x^3 - 6x^2 + 8$
 $= x^4 + x^3 - 6x^2 - 4x + 8$
- (4) $(x^3 - 3 + 4x^2)(2 + x^2)$
 $= (x^3 + 4x^2 - 3)(x^2 + 2)$
 $= (x^3 + 4x^2 - 3)x^2 + (x^3 + 4x^2 - 3) \cdot 2$
 $= x^5 + 4x^4 - 3x^2 + 2x^3 + 8x^2 - 6$
 $= x^5 + 4x^4 + 2x^3 + 5x^2 - 6$
- (5) $(x + y)(x^2 - xy + 2y^2)$
 $= x(x^2 - xy + 2y^2) + y(x^2 - xy + 2y^2)$
 $= x^3 - x^2y + 2xy^2 + x^2y - xy^2 + 2y^3$
 $= x^3 + xy^2 + 2y^3$
- (6) $(2x - 3y + 1)(x + y - 2)$
 $= (2x - 3y + 1)x + (2x - 3y + 1)y + (2x - 3y + 1) \cdot (-2)$
 $= 2x^2 - 3xy + x + 2xy - 3y^2 + y - 4x + 6y - 2$
 $= 2x^2 - xy - 3y^2 - 3x + 7y - 2$

2 [数学 I p12 練習10]

- (1) $(2x + 3)^2 = 4x^2 + 12x + 9$
- (2) $(2x - 5y)^2 = 4x^2 - 20xy + 25y^2$
- (3) $(4x + 3)(4x - 3) = 16x^2 - 9$
- (4) $(x + 2)(x + 8) = x^2 + 10x + 16$
- (5) $(x - 4)(x - 3) = x^2 - 7x + 12$
- (6) $(x - 3y)(x + 5y) = x^2 + 2xy - 15y^2$

3 [数学 I p13 練習11]

- (1) $(2x + 3)(6x + 5) = 12x^2 + 28x + 15$
- (2) $(5x + 2)(3x - 8) = 15x^2 - 34x - 16$
- (3) $(2x - y)(x + 3y) = 2x^2 + 5xy - 3y^2$
- (4) $(3x - a)(4x - 5a) = 12x^2 - 19ax + 5a^2$

4 [数学 I p14 練習12]

- (1) $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
- (2) $(x - 2y + z)^2 = x^2 + 4y^2 + z^2 - 4xy - 4yz + 2zx$

5 [数学 I p14 練習13]

- (1) $(a + b)^2(a - b)^2$
 $= \{(a + b)(a - b)\}^2$
 $= (a^2 - b^2)^2$
 $= a^4 - 2a^2b^2 + b^4$
- (2) $(x^2 + 1)(x + 1)(x - 1)$
 $= (x^2 + 1)(x^2 - 1)$
 $= (x^2)^2 - 1$
 $= x^4 - 1$
- (3) $(x^2 + 2x + 3)(x^2 - 2x + 3)$
 $= \{(x^2 + 3) + 2x\}\{(x^2 + 3) - 2x\}$
 $= (x^2 + 3)^2 - (2x)^2$
 $= x^4 + 6x^2 + 9 - 4x^2$
 $= x^4 + 2x^2 + 9$
- (4) $(x - y + z)(x + y - z)$
 $= \{x - (y - z)\}\{x + (y - z)\}$
 $= x^2 - (y - z)^2$
 $= x^2 - (y^2 - 2yz + z^2)$
 $= x^2 - y^2 - z^2 + 2yz$

6 [数学 I p15 練習14]

- (1) $2x^2y - 6xy^2 + 10xyz$
 $= 2xy \cdot x - 2xy \cdot 3y + 2xy \cdot 5z$
 $= 2xy(x - 3y + 5z)$
- (2) $4xy^2z - x^2yz^2 + 2xyz$
 $= xyz \cdot 4y - xyz \cdot xz + xyz \cdot 2$
 $= xyz(4y - xz + 2)$
- (3) $a(x - y) - bx + by$
 $= a(x - y) - b(x - y)$
 $= (x - y)(a - b)$
- (4) $y(5x - 3) + 2(3 - 5x)$
 $= y(5x - 3) - 2(5x - 3)$
 $= (5x - 3)(y - 2)$

7 [数学 I p16 練習15]

- (1) $x^2 - 8x + 16$
 $= x^2 - 2 \cdot x \cdot 4 + 4^2$
 $= (x - 4)^2$
- (2) $4x^2 + 28xy + 49y^2$
 $= (2x)^2 + 2 \cdot 2x \cdot 7y + (7y)^2$
 $= (2x + 7y)^2$
- (3) $9a^2 - 48ab + 64b^2$
 $= (3a)^2 - 2 \cdot 3a \cdot 8b + (8b)^2$
 $= (3a - 8b)^2$
- (4) $16x^2 - 25y^2$
 $= (4x)^2 - (5y)^2$
 $= (4x + 5y)(4x - 5y)$
- (5) $x^2 + 6x + 8$
 $= x^2 + (2 + 4)x + 2 \cdot 4$
 $= (x + 2)(x + 4)$
- (6) $x^2 - 5xy + 6y^2$
 $= x^2 + (-2y - 3y)x + (-2y) \cdot (-3y)$
 $= (x - 2y)(x - 3y)$
- (7) $x^2 + xy - 12y^2$
 $= x^2 + (-3y + 4y)x + (-3y) \cdot 4y$
 $= (x - 3y)(x + 4y)$
- (8) $x^2 - 2ax - 15a^2$
 $= x^2 + (3a - 5a)x + 3a \cdot (-5a)$
 $= (x + 3a)(x - 5a)$

8 [数学 I 練習16]

(1) $2x^2 + 3x + 1 = (x+1)(2x+1)$

(2) $4x^2 - 15x + 9 = (x-3)(4x-3)$

(3) $6x^2 - 5x - 6 = (2x-3)(3x+2)$

(4) $3x^2 - 2xy - y^2 = (x-y)(3x+y)$

(5) $3a^2 - 14ab + 8b^2 = (a-4b)(3a-2b)$

(6) $4x^2 + 7ax - 2a^2 = (x+2a)(4x-a)$

9 [数学 I 練習17]

(1) $x^2 - y^2 + 6y - 9$
 $= x^2 - (y^2 - 6y + 9)$
 $= x^2 - (y-3)^2$
 $= \{x + (y-3)\}\{x - (y-3)\}$
 $= (x+y-3)(x-y+3)$

(2) $x^2 - 4x + 4 - 9y^2$
 $= (x^2 - 4x + 4) - 9y^2$
 $= (x-2)^2 - (3y)^2$
 $= \{(x-2) + 3y\}\{(x-2) - 3y\}$
 $= (x+3y-2)(x-3y-2)$

10 [数学 I 練習18]

(1) $x^4 - 5x^2 + 4$
 $= (x^2)^2 - 5x^2 + 4$
 $= (x^2 - 1)(x^2 - 4)$
 $= (x+1)(x-1)(x+2)(x-2)$

(2) $x^4 - 81$
 $= (x^2)^2 - 9^2$
 $= (x^2 + 9)(x^2 - 9)$
 $= (x^2 + 9)(x+3)(x-3)$

(3) $(x^2 + 3x)^2 - 2(x^2 + 3x) - 8$
 $= \{(x^2 + 3x) + 2\}\{(x^2 + 3x) - 4\}$
 $= (x^2 + 3x + 2)(x^2 + 3x - 4)$
 $= (x+1)(x+2)(x-1)(x+4)$

11 [チャート数学 I 例題12]

(1) $2(x-1)^2 - 11(x-1) + 15$
 $= \{(x-1) - 3\}\{2(x-1) - 5\}$
 $= (x-4)(2x-7)$

(2) $x^2 - y^2 + 4y - 4$
 $= x^2 - (y^2 - 4y + 4)$
 $= x^2 - (y-2)^2$
 $= \{x + (y-2)\}\{x - (y-2)\}$
 $= (x+y-2)(x-y+2)$

(3) $x^4 - 10x^2 + 9$
 $= (x^2)^2 - 10x^2 + 9$
 $= (x^2 - 1)(x^2 - 9)$
 $= (x+1)(x-1)(x+3)(x-3)$

(4) $(x^2 + 3x)^2 - 2(x^2 + 3x) - 8$
 $= \{(x^2 + 3x) + 2\}\{(x^2 + 3x) - 4\}$
 $= (x^2 + 3x + 2)(x^2 + 3x - 4)$
 $= (x+1)(x+2)(x-1)(x+4)$

12 [チャート数学 I 例題14]

(1) $9b^2 + 3ab - 2a - 4$
 $= (3b-2)a + 9b^2 - 4$
 $= (3b-2)a + (3b+2)(3b-2)$
 $= (3b-2)(a+3b+2)$

(2) $x^3 - x^2y - xz^2 + yz^2$
 $= (z^2 - x^2)y + x^3 - xz^2$
 $= (z^2 - x^2)y - x(z^2 - x^2)$
 $= (z^2 - x^2)(y-x)$
 $= (z+x)(z-x)(y-x)$
 $= (x-y)(x-z)(x+z)$

(3) $1 + 2ab + a + 2b$
 $= (2b+1)a + 2b + 1$
 $= (a+1)(2b+1)$

別解 $1 + 2ab + a + 2b$
 $= (2a+2)b + a + 1$
 $= 2(a+1)b + a + 1$
 $= (a+1)(2b+1)$

$$\begin{array}{r} 1 \quad \times \quad -3 \quad \longrightarrow \quad -6 \\ 2 \quad \quad \quad -5 \quad \longrightarrow \quad -5 \\ \hline 2 \quad \quad \quad 15 \quad \quad -11 \end{array}$$

13 [チャート数学 I 例題15]

(1) (与式) $= x^2 - (y+1)x - (2y^2 + 7y + 6)$
 $= x^2 - (y+1)x - (y+2)(2y+3)$

$= \{x + (y+2)\}\{x - (2y+3)\}$
 $= (x+y+2)(x-2y-3)$

(2) (与式) $= 3x^2 + (7y-5)x + (2y^2 - 5y + 2)$
 $= 3x^2 + (7y-5)x + (y-2)(2y-1)$
 $= \{x + (2y-1)\}\{3x + (y-2)\}$
 $= (x+2y-1)(3x+y-2)$

$$\begin{array}{r} 1 \quad \times \quad 2 \quad \longrightarrow \quad 4 \\ 2 \quad \quad \quad 3 \quad \longrightarrow \quad 3 \\ \hline 2 \quad \quad \quad 6 \quad \quad 7 \end{array}$$

$$\begin{array}{r} 1 \quad \times \quad \quad y+2 \quad \longrightarrow \quad y+2 \\ 1 \quad \quad \quad -(2y+3) \quad \longrightarrow \quad -2y-3 \\ \hline 1 \quad \quad \quad -(y+2)(2y+3) \quad \longrightarrow \quad -(y+1) \end{array}$$

$$\begin{array}{r} 1 \quad \times \quad -2 \quad \longrightarrow \quad -4 \\ 2 \quad \quad \quad -1 \quad \longrightarrow \quad -1 \\ \hline 2 \quad \quad \quad 2 \quad \quad -5 \end{array}$$

$$\begin{array}{r} 1 \quad \times \quad \quad 2y-1 \quad \longrightarrow \quad 6y-3 \\ 3 \quad \quad \quad y-2 \quad \longrightarrow \quad y-2 \\ \hline 3 \quad \quad \quad (y-2)(2y-1) \quad 7y-5 \end{array}$$

14 [チャート数学 I 例題16]

(1) $a^2b + ab^2 + b^2c + bc^2 + c^2a + ca^2 + 2abc$
 $= (b+c)a^2 + (b^2 + 2bc + c^2)a + b^2c + bc^2$
 $= (b+c)a^2 + (b+c)^2a + (b+c)bc$
 $= (b+c)\{a^2 + (b+c)a + bc\}$
 $= (b+c)(a+b)(a+c)$
 $= (a+b)(b+c)(c+a)$

(2) $a^2(b-c) + b^2(c-a) + c^2(a-b)$
 $= (b-c)a^2 + b^2c - ab^2 + c^2a - bc^2$
 $= (b-c)a^2 - (b^2 - c^2)a + (b-c)bc$
 $= (b-c)a^2 - (b+c)(b-c)a + (b-c)bc$
 $= (b-c)\{a^2 - (b+c)a + bc\}$
 $= (b-c)(a-b)(a-c)$
 $= -(a-b)(b-c)(c-a)$

15 [チャート数学 I 練習5] αコース

$$(1) (3x+5y)^2 = (3x)^2 + 2 \cdot 3x \cdot 5y + (5y)^2 = 9x^2 + 30xy + 25y^2$$

$$(2) (a^2+2b)^2 = (a^2)^2 + 2 \cdot a^2 \cdot 2b + (2b)^2 = a^4 + 4a^2b + 4b^2$$

$$(3) (3a-2b)^2 = (3a)^2 - 2 \cdot 3a \cdot 2b + (2b)^2 = 9a^2 - 12ab + 4b^2$$

$$(4) (2xy-3)^2 = (2xy)^2 - 2 \cdot 2xy \cdot 3 + 3^2 = 4x^2y^2 - 12xy + 9$$

$$(5) (2x-3y)(2x+3y) = (2x+3y)(2x-3y) = (2x)^2 - (3y)^2 = 4x^2 - 9y^2$$

$$(6) (3x-4y)(5y+4x) = (3x-4y)(4x+5y) = 3 \cdot 4x^2 + \{3 \cdot 5y + (-4y) \cdot 4\}x + (-4y) \cdot 5y = 12x^2 - xy - 20y^2$$

16 [チャート数学 I 練習6] αコース

$$(1) (x+2)(x^2-2x+4) = (x+2)(x^2-x \cdot 2+2^2) = x^3+2^3 = x^3+8$$

$$(2) (2p-q)(4p^2+2pq+q^2) = (2p-q)\{(2p)^2+2p \cdot q+q^2\} = (2p)^3-q^3 = 8p^3-q^3$$

$$(3) (2x+1)^3 = (2x)^3 + 3 \cdot (2x)^2 \cdot 1 + 3 \cdot 2x \cdot 1^2 + 1^3 = 8x^3 + 12x^2 + 6x + 1$$

$$(4) (3x-2y)^3 = (3x)^3 - 3 \cdot (3x)^2 \cdot 2y + 3 \cdot 3x \cdot (2y)^2 - (2y)^3 = 27x^3 - 54x^2y + 36xy^2 - 8y^3$$

17 [チャート数学 I 練習7] αコース

$$(1) (a+3b-c)^2 = \{a+3b+(-c)\}^2 = a^2 + (3b)^2 + (-c)^2 + 2 \cdot a \cdot 3b + 2 \cdot 3b(-c) + 2(-c)a = a^2 + 9b^2 + c^2 + 6ab - 6bc - 2ca$$

$$(2) (x+y+7)(x+y-7) = \{(x+y)+7\}\{(x+y)-7\} = (x+y)^2 - 7^2 = x^2 + 2xy + y^2 - 49$$

$$(3) (x-3y+2z)(x+3y-2z) = \{x-(3y-2z)\}\{x+(3y-2z)\} = x^2 - (3y-2z)^2 = x^2 - 9y^2 - 4z^2 + 12yz$$

$$(4) (x^2-3x+1)(x^2+4x+1) = \{(x^2+1)-3x\}\{(x^2+1)+4x\} = (x^2+1)^2 + (x^2+1)x - 12x^2 = (x^4+2x^2+1) + x^3+x - 12x^2 = x^4+x^3-10x^2+x+1$$

18 [チャート数学 I 練習8] αコース

$$(1) (x+3)(x-3)(x^2+9) = (x^2-9)(x^2+9) = (x^2)^2 - 9^2 = x^4 - 81$$

$$(2) (x-1)(x-2)(x+1)(x+2) = (x-1)(x+1) \times (x-2)(x+2) = (x^2-1) \times (x^2-4) = (x^2)^2 - 5x^2 + 4 = x^4 - 5x^2 + 4$$

$$(3) (a+b)^3(a-b)^3 = \{(a+b)(a-b)\}^3 = (a^2-b^2)^3 = (a^2)^3 - 3(a^2)^2b^2 + 3a^2(b^2)^2 - (b^2)^3 = a^6 - 3a^4b^2 + 3a^2b^4 - b^6$$

$$(4) (x+3)(x-1)(x^2+x+1)(x^2-3x+9) = (x-1)(x^2+x+1) \times (x+3)(x^2-3x+9) = (x^3-1)(x^3+27) = (x^3)^2 + 26x^3 - 27 = x^6 + 26x^3 - 27$$

19 [チャート数学 I 練習10] αコース

$$(1) (a+b)x - (a+b)y = (a+b)(x-y)$$

$$(2) (a-b)x^2 + (b-a)xy = (a-b)x^2 - (a-b)xy = x(a-b)(x-y)$$

$$(3) 121 - 49x^2y^2 = 11^2 - (7xy)^2 = (11+7xy)(11-7xy) = -(7xy+11)(7xy-11)$$

$$(4) 8xyz^2 - 40xyz + 50xy = 2xy(4z^2 - 20z + 25) = 2xy\{(2z)^2 - 2 \cdot 2z \cdot 5 + 5^2\} = 2xy(2z-5)^2$$

$$(5) x^2 - 8x + 12 = x^2 + (-2-6) \cdot x + (-2) \cdot (-6) = (x-2)(x-6)$$

$$(6) a^2 + 5ab - 150b^2 = a^2 + (15b-10b) \cdot a + 15b \cdot (-10b) = (a+15b)(a-10b)$$

$$(7) x^2 - xy - 12y^2 = x^2 + (3y-4y) \cdot x + 3y \cdot (-4y) = (x+3y)(x-4y)$$

$$(8) 3x^2 + 10x + 3 = (x+3)(3x+1) \quad \begin{array}{r} 1 \times 3 \rightarrow 9 \\ 3 \times 1 \rightarrow 3 \\ \hline 3 \quad 3 \quad 10 \end{array}$$

$$(9) 2x^2 - 9x + 4 = (x-4)(2x-1) \quad \begin{array}{r} 1 \times -4 \rightarrow -4 \\ 2 \times -1 \rightarrow -2 \\ \hline 2 \quad 4 \quad -9 \end{array}$$

$$(10) 6x^2 + x - 1 = (2x+1)(3x-1) \quad \begin{array}{r} 2 \times 1 \rightarrow 2 \\ 3 \times -1 \rightarrow -3 \\ \hline 6 \quad -1 \quad 1 \end{array}$$

$$(11) 8x^2 - 2xy - 3y^2 = (2x+y)(4x-3y) \quad \begin{array}{r} 2 \times y \rightarrow 2y \\ 4 \times -3y \rightarrow -12y \\ \hline 8 \quad -3y^2 \quad -2y \end{array}$$

$$(12) 6a^2 - ab - 12b^2 = (2a-3b)(3a+4b) \quad \begin{array}{r} 2 \times -3b \rightarrow -6b \\ 3 \times 4b \rightarrow 12b \\ \hline 6 \quad -12b^2 \quad -b \end{array}$$

$$(13) 10p^2 - 19pq + 6q^2 = (2p-3q)(5p-2q) \quad \begin{array}{r} 2 \times -3q \rightarrow -6q \\ 5 \times -2q \rightarrow -10q \\ \hline 10 \quad 6q^2 \quad -19q \end{array}$$

20 [チャート数学 I 練習11] α コース

- (1) $8a^3 + 27b^3 = (2a)^3 + (3b)^3$
 $= (2a + 3b)\{(2a)^2 - 2a \cdot 3b + (3b)^2\}$
 $= (2a + 3b)(4a^2 - 6ab + 9b^2)$
- (2) $64x^3 - 1 = (4x)^3 - 1^3 = (4x - 1)\{(4x)^2 + 4x \cdot 1 + 1^2\}$
 $= (4x - 1)(16x^2 + 4x + 1)$
- (3) $8x^3 - 36x^2 + 54x - 27 = (2x)^3 - 3 \cdot (2x)^2 \cdot 3 + 3 \cdot 2x \cdot 3^2 - 3^3$
 $= (2x - 3)^3$

別解 $8x^3 - 36x^2 + 54x - 27 = 8x^3 - 27 - (36x^2 - 54x)$
 $= (2x - 3)(4x^2 + 6x + 9) - 18x(2x - 3)$
 $= (2x - 3)(4x^2 + 6x + 9 - 18x)$
 $= (2x - 3)(4x^2 - 12x + 9)$
 $= (2x - 3)(2x - 3)^2$
 $= (2x - 3)^3$

(4) $4x^3 - 8x^2 - 9x + 18 = 4x^2(x - 2) - 9(x - 2)$
 $= (x - 2)(4x^2 - 9)$
 $= (x - 2)(2x + 3)(2x - 3)$

15 [チャート数学 I 例題13] β コース

- (1) $(x^2 + x - 5)(x^2 + x - 7) + 1 = \{(x^2 + x) - 5\}\{(x^2 + x) - 7\} + 1$
 $= (x^2 + x)^2 - 12(x^2 + x) + 36$
 $= (x^2 + x - 6)^2$
 $= \{(x + 3)(x - 2)\}^2$
 $= (x + 3)^2(x - 2)^2$
- (2) $(x + 1)(x + 2)(x + 3)(x + 4) - 24 = \{(x + 1)(x + 4)\} \times \{(x + 2)(x + 3)\} - 24$
 $= \{(x^2 + 5x) + 4\}\{(x^2 + 5x) + 6\} - 24$
 $= (x^2 + 5x)^2 + 10(x^2 + 5x)$
 $= (x^2 + 5x)(x^2 + 5x + 10)$
 $= x(x + 5)(x^2 + 5x + 10)$
- (3) (与式) $= \{(x + y)^2\}^2 - \{(x - y)^2\}^2$
 $= \{(x + y)^2 + (x - y)^2\}\{(x + y)^2 - (x - y)^2\}$
 $= (2x^2 + 2y^2)(2xy + 2xy)$
 $= 2(x^2 + y^2) \cdot 4xy$
 $= 8xy(x^2 + y^2)$

16 [チャート数学 I 例題18] β コース

- (1) $x^4 + 4x^2 + 16 = (x^4 + 8x^2 + 16) - 4x^2$
 $= (x^2 + 4)^2 - (2x)^2$
 $= \{(x^2 + 4) + 2x\}\{(x^2 + 4) - 2x\}$
 $= (x^2 + 2x + 4)(x^2 - 2x + 4)$
- (2) $x^4 - 7x^2y^2 + y^4 = (x^4 + 2x^2y^2 + y^4) - 9x^2y^2$
 $= (x^2 + y^2)^2 - (3xy)^2$
 $= \{(x^2 + y^2) + 3xy\}\{(x^2 + y^2) - 3xy\}$
 $= (x^2 + 3xy + y^2)(x^2 - 3xy + y^2)$

(3) $4x^4 + 1 = (4x^4 + 4x^2 + 1) - 4x^2$
 $= (2x^2 + 1)^2 - (2x)^2$
 $= \{(2x^2 + 1) + 2x\}\{(2x^2 + 1) - 2x\}$
 $= (2x^2 + 2x + 1)(2x^2 - 2x + 1)$

17 [チャート数学 I EXERCISES9] β コース

(1) $x^2 - 2xy + y^2 - x + y = (x^2 - 2xy + y^2) - (x - y)$
 $= (x - y)^2 - (x - y)$
 $= (x - y)(x - y - 1)$

別解 (与式) $= x^2 - (2y + 1)x + y(y + 1)$
 $= (x - y)(x - y - 1)$

(2) $81x^4 - y^4 = (9x^2)^2 - (y^2)^2$
 $= (9x^2 + y^2)(9x^2 - y^2)$
 $= (3x + y)(3x - y)(9x^2 + y^2)$

(3) $4x^4 - 37x^2y^2 + 9y^4 = 4(x^2)^2 - 37y^2 \cdot x^2 + 9y^4$
 $= (x^2 - 9y^2)(4x^2 - y^2)$
 $= (x + 3y)(x - 3y)(2x + y)(2x - y)$

1	×	-9y ²	→	-36y ²
4		-y ²	→	-y ²
4		9y ⁴		-37y ²

(4) $x^2 - x = X$ とおくと, $-8x^2 + 8x = -8X$ であるから
 $(x^2 - x)^2 - 8x^2 + 8x + 12 = X^2 - 8X + 12$
 $= (X - 2)(X - 6)$
 $= (x^2 - x - 2)(x^2 - x - 6)$
 $= (x + 1)(x - 2)(x + 2)(x - 3)$

18 [チャート数学 I EXERCISES10] β コース

(1) $x^6 - 1 = (x^3)^2 - 1$
 $= (x^3 + 1)(x^3 - 1)$
 $= (x + 1)(x^2 - x + 1)(x - 1)(x^2 + x + 1)$
 $= (x + 1)(x - 1)(x^2 - x + 1)(x^2 + x + 1)$

(2) $(x + y)^6 - (x - y)^6 = \{(x + y)^3\}^2 - \{(x - y)^3\}^2$
 $= \{(x + y)^3 + (x - y)^3\}\{(x + y)^3 - (x - y)^3\}$
 $= (x^3 + 3x^2y + 3xy^2 + y^3 + x^3 - 3x^2y + 3xy^2 - y^3)$
 $\times (x^3 + 3x^2y + 3xy^2 + y^3 - x^3 + 3x^2y - 3xy^2 + y^3)$
 $= (2x^3 + 6xy^2)(6x^2y + 2y^3)$
 $= 2x(x^2 + 3y^2) \cdot 2y(3x^2 + y^2)$
 $= 4xy(x^2 + 3y^2)(3x^2 + y^2)$

(3) $x^6 - 19x^3 - 216 = (x^3)^2 - 19x^3 - 216$
 $= (x^3 + 8)(x^3 - 27)$
 $= (x + 2)(x^2 - 2x + 4)(x - 3)(x^2 + 3x + 9)$
 $= (x + 2)(x - 3)(x^2 - 2x + 4)(x^2 + 3x + 9)$

(4) $x^6 - 2x^3 + 1 = (x^3)^2 - 2x^3 + 1$
 $= (x^3 - 1)^2$
 $= \{(x - 1)(x^2 + x + 1)\}^2$
 $= (x - 1)^2(x^2 + x + 1)^2$

19 [チャート数学 I EXERCISES11] βコース

(1) (与式) $= (2x+5y)((2x+5y)+8) - 65$
 $= (2x+5y)^2 + 8(2x+5y) - 65$
 $= \{(2x+5y)-5\}\{(2x+5y)+13\}$
 $= (2x+5y-5)(2x+5y+13)$

(2) $x+3y=X$ とおくと

(与式) $= (X-1)(X+3)(X+4) + 12 = (X-1)(X^2+7X+12) + 12$
 $= X^3+7X^2+12X-X^2-7X-12+12$
 $= X^3+6X^2+5X = X(X^2+6X+5) = X(X+1)(X+5)$
 $= (x+3y)(x+3y+1)(x+3y+5)$

(3) $2x-3=X$ とおくと, $2x+1=X+4$ であるから

(与式) $= 3X^2 - 4(X+4) + 12 = 3X^2 - 4X - 4$
 $= (X-2)(3X+2) = (2x-3-2)\{3(2x-3)+2\}$
 $= (2x-5)(6x-7)$

$$\begin{array}{r} 1 \quad \times \quad -2 \rightarrow -6 \\ 3 \quad \quad 2 \rightarrow 2 \\ \hline 3 \quad -4 \quad -4 \end{array}$$

(4) (与式) $= 2(x+1)^4 + 5(x+1)^2(x-1)^2 + 2(x-1)^4$

ここで, $(x+1)^2 = a$, $(x-1)^2 = b$ とおくと

(与式) $= 2a^2 + 5ab + 2b^2 = (a+2b)(2a+b)$
 $= \{(x+1)^2 + 2(x-1)^2\}\{2(x+1)^2 + (x-1)^2\}$
 $= (3x^2 - 2x + 3)(3x^2 + 2x + 3)$

$$\begin{array}{r} 1 \quad \times \quad 2 \rightarrow 4 \\ 2 \quad \quad 1 \rightarrow 1 \\ \hline 2 \quad 2 \quad 5 \end{array}$$

(5) (与式) $= \{(x+1)(x+4)\}\{(x+2)(x+3)\} + 1$

$= (x^2+5x+4)(x^2+5x+6) + 1$
 $= (x^2+5x)^2 + 10(x^2+5x) + 25$
 $= \{(x^2+5x)+5\}^2$
 $= (x^2+5x+5)^2$

20 [チャート数学 I EXERCISES13] βコース

(1) $x^2y - 2xyz - y - xy^2 + x - 2z$
 $= -2(xy+1)z + x^2y - xy^2 + x - y$
 $= -2(xy+1)z + xy(x-y) + (x-y)$
 $= -2(xy+1)z + (x-y)(xy+1)$
 $= (xy+1)(x-y-2z)$

(2) $8x^3 + 12x^2y + 4xy^2 + 6x^2 + 9xy + 3y^2$
 $= (4x+3)y^2 + (12x^2+9x)y + 8x^3 + 6x^2$
 $= (4x+3)y^2 + 3x(4x+3)y + 2x^2(4x+3)$
 $= (4x+3)(y^2+3xy+2x^2)$
 $= (4x+3)(y+x)(y+2x)$
 $= (4x+3)(x+y)(2x+y)$

(3) $x^3y + x^2y^2 + x^3 + x^2y - xy - y^2 - x - y$
 $= (x^2-1)y^2 + (x^3+x^2-x-1)y + x^3 - x$
 $= (x^2-1)y^2 + \{x(x^2-1) + x^2-1\}y + x(x^2-1)$
 $= (x^2-1)y^2 + (x+1)(x^2-1)y + x(x^2-1)$
 $= (x^2-1)\{y^2 + (x+1)y + x\}$
 $= (x+1)(x-1)(y+x)(y+1)$
 $= (x+1)(x-1)(x+y)(y+1)$

別解 $x^3y + x^2y^2 + x^3 + x^2y - xy - y^2 - x - y$
 $= x^2y(x+y) + x^2(x+y) - y(x+y) - (x+y)$
 $= (x+y)(x^2y + x^2 - y - 1)$
 $= (x+y)\{(x^2-1)y + x^2 - 1\}$
 $= (x+y)(x^2-1)(y+1)$
 $= (x+y)(x+1)(x-1)(y+1)$